DO-003-1164003 Seat No. ______

M. Sc. (Sem. IV) Examination

March / April - 2022

Mathematics : CMT-4003

(Number Theory-2)

Faculty Code : 003
Subject Code : 1164003

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) There are five questions.
(2) All the questions are compulsory.
(3) Each question carries 14 marks.

1 Do as directed : (answer any **seven**) **14**

(a) If $x = < 2, 2, 2, 2, ... >$ then, $x^2 =$ _______.

(b) If n is the smallest positive integer such that $\frac{14}{349}$ appears in the n^{th} row then $n =$ ________.

(c) Prove that, $h_n k_{n-2} - h_{n-2} k_n = (-1)^n a_n$.

(d) If (x, y, z) is a Pythagorean Triplet show that, $\gcd(x, z) = \gcd(y, z)$.

(e) Show that, there are infinitely many solutions (x, y) of $x^2 - dy^2 = 1$ in which k/y for d >1 is not a perfect square and $k \geq 1$.

(f) Express $\frac{2022}{29}$ and $\frac{1992}{5}$ in contined fraction expansion.

(g) Find the value of <1,3,3,3,3,..., ...> and <–1,2,3,4,5>.

(h) Find the irrational number having continued fraction expansion $< 8, \overline{1, 16} >$.

(i) Define : (i) Periodic continued fraction form and (ii) Primitive pythagorean triplet with examples.

(j) Write down any two solution of $3x + 5y = 8$.

2 Answer any **two** of the following : **14**

(1) Prove that, π is an irrational.

(2) State and prove, Hurwitz Inequality for Farey Fractions.

(3) State and prove, necessary and sufficient condition under which the linear diophantine equation has a solution.

3 Answer the following : **14**

(1) Prove that, if θ is an irrational number then there are infinitely many rationales $\frac{a}{b}$ such that $\left|\theta - \frac{a}{b}\right| < \frac{1}{b^2}; b > 0$ and $(a, b) = 1$.

(2) If the continued fraction expansion of an irrational number x is periodic then prove that, the number x is quadratic irrational.

OR

3 Answer the following : **14**

(1) Suppose $\frac{x}{y}$ is a rational number where y is positive and $(x, y) = 1$. Prove that, $\frac{x}{y}$ appears in the y^{th} row of farey fractions and subsequent rows.

(2) Suppose $C_n x^n + C_{n-1} x^{n-1} + \cdots + C_0$ is a polynomial with integer co-efficient and $\frac{s}{t}$ is a rational number with t is a positive and $(s, t) = 1$. If $\frac{s}{t}$ is a root of this polynomial then prove that, s divide C_0 and t divide to C_n. Hence deduce that, if a is an integer and $x^n = a$ has a rational root then it must be an integer.

4 Answer the following : **14**

(1) (i) Let $\frac{a}{b}$ and $\frac{c}{d}$ be consecutive farey fractions in the n^{th} row. Show that, $|ad - bc| = 1$.

(ii) Show that, $15x^2 - 7y^2 = 9$ does not have a solution in integers.

(2) (i) Prove that, there are infinitely many positive integers n such that $n^2 + (n+1)^2$ is a perfect square.

(ii) Prove or disprove that, if $a_o, a_1, a_2, a_3, ..., a_n$ is a sequence of integers for $a_i \geq 1; \forall\, i \geq 1$ with $\theta = \lim_{n \to \infty} r_{2n}$ then $\theta = \lim_{n \to \infty} r_n$ where $r_n = \frac{h_n}{k_n}$.

5 Answer any **two** of the following : **14**

(1) Show that, if the triplet (x, y, z) is a primitive pythagorean triplet then there exists r and s such that $r > s \geq 1, (r, s) = 1$ and r is even then s is odd and vice-versa.

(2) Show that, the equation $a^4 + b^4 = c^2$ has no solution in integers.

(3) Prove that, if θ is an irrational number and suppose for some rational number $\frac{a}{b}$ for b>0 and (a, b)=1 with $|\theta b - a| < |\theta k_n - h_n|$ for some n then $b \geq k_{n+1}$.

(4) Show that, the equation $x^2 = y^3 + 7$ has no solution in integers.